# Rules for Differentiating

If f & g are both differentiable functions, then:

1. The Sum Rule

$$\frac{d}{dx}\left[f(x)+g(x)\right] = f'(x) + g'(x)$$

$\boxed{Ex}$ $y = x^3 + 6x$ $\quad y' = 3x^2 + 6$

2. The Difference Rule

$$\frac{d}{dx}\left[f(x)-g(x)\right] = f'(x) - g'(x)$$

$\boxed{Ex}$ $y = x^4 - e^x$ $\quad y' = 4x^3 - e^x$

# Rules for Differentiating (continued)

If f & g are both differentiable functions, then:

3. The Product Rule

$$\frac{d}{dx}\left[f(x)\cdot g(x)\right] = \underbrace{f(x)}_{\text{first funct.}}\cdot\underbrace{g'(x)}_{\text{Deriv. of 2nd funct.}} + \underbrace{g(x)}_{\text{2nd funct.}}\cdot\underbrace{f'(x)}_{\text{Deriv. of 1st func.}}$$

$\boxed{Ex}$ $y = xe^x$

Notice that our first function is $x$, and the second function is $e^x$.

Let's try a few examples with the Product Rule:

1. $f(x) = xe^x$

Let's find f \`(x), f \`\`(x) and f \`\`\`(x)…

$$f'(x) = xe^x + e^x \cdot 1$$
$$= xe^x + e^x$$
$$= \underbrace{e^x}\underbrace{(x+1)}$$

$$f''(x) = e^x \cdot 1 + (x+1)e^x$$
$$= e^x + e^x(x+1)$$
$$= e^x(1+x+1)$$
$$= \underbrace{e^x}\underbrace{(x+2)}$$

$$f'''(x) = e^x \cdot 1 + (x+2)e^x$$
$$= e^x + e^x(x+2)$$
$$= e^x[1+x+2]$$
$$= e^x(x+3)$$

Let's try a few examples with the Product Rule:

2. $f(x) = x^3 - 3x + x^2e^x$ Find $f`(x)$

$$f'(x) = 3x^2 - 3 + x^2e^x + e^x \cdot 2x$$
$$= 3x^2 - 3 + x^2e^x + 2xe^x$$

Let's try a few examples with the Product Rule:

3. $f(x) = \left(\sqrt{x}\right) g(x)$ where g(4) = 2 and g `(4) = 3

Find f `(4)

$$f(x) = x^{\frac{1}{2}} \cdot g(x)$$

$$f'(x) = x^{\frac{1}{2}} \cdot g'(x) + g(x) \cdot \frac{1}{2} x^{-\frac{1}{2}}$$

$$= \sqrt{x} \cdot g'(x) + \frac{1}{2\sqrt{x}} \cdot g(x)$$

$$f'(4) = \sqrt{4} \cdot 3 + \frac{1}{2\sqrt{4}} \cdot 2$$

$$= 2 \cdot 3 + \frac{1}{4} \cdot 2$$

$$= 6 + \frac{1}{2}$$

$$= 6\frac{1}{2}$$

Let's try a few examples with the Product Rule:

4. $f(t) = \left(\sqrt{t}\right)(a+bt)$ for some constants a & b

Find f \`(t)

$$f'(t) = \sqrt{t} \cdot b + (a+bt) \cdot \frac{1}{2\sqrt{t}}$$

$$= b\sqrt{t} + \frac{a+bt}{2\sqrt{t}}$$

## Rules for Differentiating (continued)

If f & g are both differentiable functions, then:

4. The Quotient Rule

$\frac{f(x)}{g(x)}$ * where $g(x) \neq 0$

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x) \cdot f'(x) - f(x) \cdot g'(x)}{[g(x)]^2}$$

$$\frac{\text{Denom} * \text{Deriv. of Num} - \text{Num} * \text{Deriv. of Denom}}{(\text{Denom})^2}$$

Let's try a few examples with the Quotient Rule:

1. $f(x) = \dfrac{1-x}{2+x}$ Find f `(x)

$$f'(x) = \frac{(2+x)\cdot -1 - (1-x)\cdot 1}{(2+x)^2}$$

$$= \frac{-2-x-1+x}{(2+x)^2} = \frac{-3}{(2+x)^2}$$

Let's try a few examples with the Quotient Rule:

2. $f(x) = \dfrac{e^x}{x^2 - 3}$ Find f `(x)

$$f'(x) = \frac{(x^2-3)\cdot e^x - e^x \cdot 2x}{(x^2-3)^2}$$

$$= \boxed{\frac{(x^2-3)\cdot e^x - 2xe^x}{(x^2-3)^2}}$$

Let's try a few examples with the Quotient Rule:

3. $f(x) = \dfrac{x^2 + x - 2}{x^3 + 6}$ Find f `(x)

$$f'(x) = \frac{(x^3+6)(2x+1) - (x^2+x-2)\cdot 3x^2}{(x^3+6)^2}$$

Let's try a few examples with the Quotient Rule:

4. $f(x) = \dfrac{3x^2 + 2\sqrt{x}}{x}$ Find f `(x)

$f(x) = \dfrac{3x^2}{x} + \dfrac{2\sqrt{x}}{x} = 3x + 2x^{-\frac{1}{2}}$

$f'(x) = 3 - x^{-\frac{3}{2}}$

Let's try a few examples with the Quotient Rule:

5. Find the equation of the tangent line to the curve $y = \frac{e^x}{1+x^2}$ at the point $\left(1, \frac{1}{2}e\right)$

$$y' = \frac{(1+x^2)e^x - e^x \cdot 2x}{(1+x^2)^2} = \frac{(1+x^2) \cdot e^x - 2xe^x}{(1+x^2)^2}$$

$$m = y'(1) = \frac{(1+1^2) \cdot e^1 - 2(1)e^1}{(1+1^2)^2} = \frac{2e^1 - 2e^1}{4} = \frac{0}{4} = 0$$

$$y - \frac{1}{2}e = 0(x-1)$$

$$y = \frac{1}{2}e$$

Homework

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